

## Introduction

### Graph signal processing

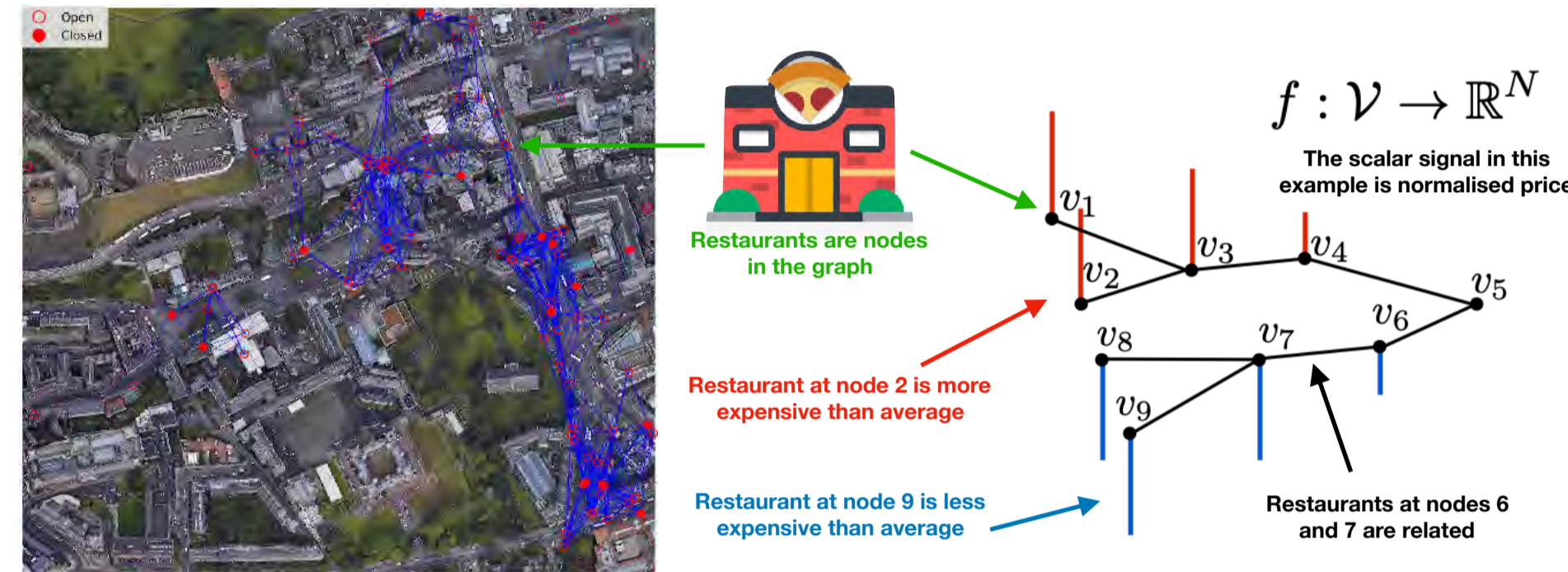
- Processing of signals defined on vertex set of weighted and undirected graphs

### Geometric deep learning

- Deep learning on data residing on graphs
- Spectral approaches take inspiration from graph signal processing

### Business classification

- Given attributes of a business and a graph topology relating businesses in Edinburgh, can we predict if the restaurant has closed down?
- We use a graph convolutional neural network for classification hopefully exploiting pairwise relationships between businesses



## Graph CNN framework

### Filtering in the graph spectral domain

- Eigenvalue decomposition is  $\mathcal{O}(n^3)$
- Change of basis is also  $\mathcal{O}(n^3)$
- Can approximate filtering using Chebyshev polynomials [1]
- Further simplification by considering polynomials of order one [2]

### Propagation rule:

$$H^{(l+1)} = \text{ReLU} \left( \tilde{D}^{-\frac{1}{2}} \tilde{A} \tilde{D}^{-\frac{1}{2}} H^{(l)} W^{(l)} \right)$$

Degree matrix

Adjacency matrix

Parameter matrix

### Layer properties - CNN correspondence

- Signal value on node  $i$  in layer  $K+1$  is a function of the value at  $i$  as well as that at  $i$ 's 1-hop neighbours in layer  $K$  - **localised filters**
- Weight sharing across nodes - **filters independent of input size**
- Sparse multiplications give low computational cost

## Results

### Results

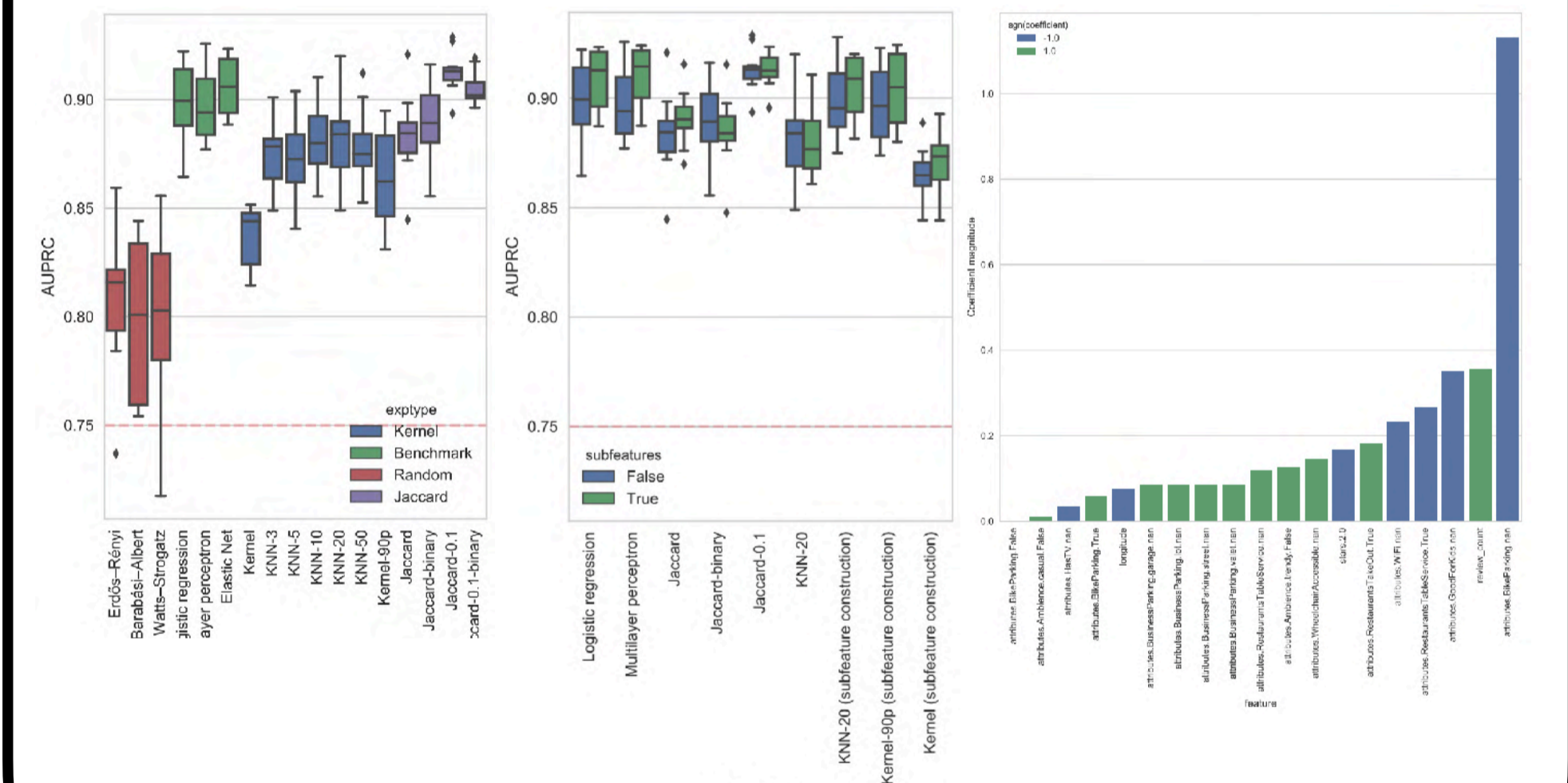
- Benchmarks were competitive
- Review data gave slight advantage when using full set of features
- Random graphs were detrimental to performance

### Feature selection

- Using elastic net selected features improved performance for many approaches
- The benefit is less significant in graph convolutional neural networks

### Thresholding edges

- Thresholding the Jaccard graph improved performance despite disconnecting the graph and isolating 10.6% of the nodes



## Graph signal processing and convolution on graphs

Laplace operator:  $-\nabla^2$

Eigenfunction basis:  $e^{i\omega x}$

Classical FT:

$$\hat{f}(\omega) = \langle e^{i\omega x}, f \rangle = \int e^{i\omega x} f(x) dx$$

$$f(x) = \frac{1}{2\pi} \int \hat{f}(\omega) e^{i\omega x} d\omega$$

Classical convolution:

$$(f * h)(x) = \int f(\tau) h(x - \tau) d\tau = \frac{1}{2\pi} \int \hat{f}(\omega) \hat{h}(\omega) e^{i\omega x} d\omega$$

Graph Laplacian:  $L = D - W$

Eigenvector basis:  $\chi_\ell$

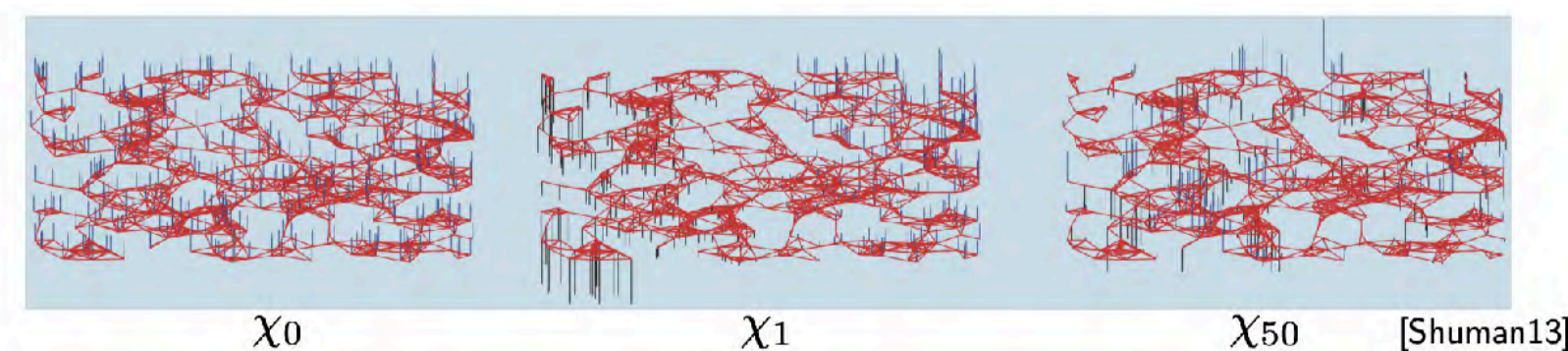
Graph FT:

$$\hat{f}(\ell) = \langle \chi_\ell, f \rangle = \sum_{i=1}^N \chi_\ell(i) f(i)$$

$$f(i) = \sum_{\ell=1}^N \hat{f}(\ell) \chi_\ell(i)$$

Graph convolution:

$$(f * h)(i) = \sum_{\ell=1}^N \hat{f}(\ell) \hat{h}(\ell) \chi_\ell(i)$$



Low frequency

High frequency

$$L = \chi \Lambda \chi^T$$

$$\chi_0^T L \chi_0 = \lambda_0 = 0$$

$$\chi_{50}^T L \chi_{50} = \lambda_{50}$$

## Classification problem

### Problem set up

- Yelp dataset - multimodal data about businesses
- Each restaurant is represented by a node in the graph with edges capturing relationship between businesses
- Each restaurant has an associated feature vector
- Label is if the restaurant is still operating (binary classification)
- Elastic net model used to select subset of features



(a) Nodes

(b) As the crow flies

(c) Jaccard

(d) Kernel

### Importance of graph topology

- Edges provide a relational inductive bias
- Using no edges is equivalent to a multilayer perceptron
- We experiment with different ways to generate edge weights:
  - Randomly
  - Kernel-based methods (e.g., Gaussian RBF kernel)
  - Restaurant distance
- Reviewer data - Jaccard index between two sets of reviewers (one for each restaurant)

## Discussion

### Graph constructions

- Social data
- Statistical methods

### Longitudinal prediction

- Can we predict business closure in the future?

### Other frameworks

- Explore alternative graph convolutional neural network frameworks

### Theoretical approach to the importance of graph topology

- Theoretical bound on the output given a small perturbation of the input graph topology

Contact: [kenlay@robots.ox.ac.uk](mailto:kenlay@robots.ox.ac.uk)

[1] Defferrard, Michaël, Xavier Bresson, and Pierre Vandergheynst. "Convolutional neural networks on graphs with fast localized spectral filtering." *Advances in Neural Information Processing Systems*. 2016.

[2] Kipf, Thomas N., and Max Welling. "Semi-supervised classification with graph convolutional networks." *arXiv preprint arXiv:1609.02907* (2016).