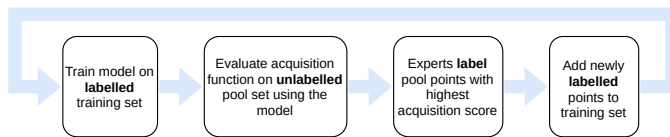


### Active Learning

A key problem in deep learning is **data efficiency**. In Active Learning, we iteratively acquire labels for only the **most informative** data points.



### BALD Acquisition Function<sup>1</sup>

We implement a Bayesian Neural Network using dropout VI<sup>2</sup> and define the acquisition function  $a$  as follows:

$$a_{\text{BALD}}(\{x_1, \dots, x_b\}, \mathbf{p}(\omega | \mathcal{D}_{\text{train}})) := \sum_{i=1}^b \mathbb{I}(y_i; \omega | x_i, \mathcal{D}_{\text{train}})$$

$$\mathbb{I}(y; \omega | x, \mathcal{D}_{\text{train}}) = \mathbb{H}(y|x, \mathcal{D}_{\text{train}}) - \mathbb{E}_{\mathbf{p}(\omega | \mathcal{D}_{\text{train}})} [\mathbb{H}(y|x, \omega, \mathcal{D}_{\text{train}})]$$

**First term** captures general uncertainty of model.

**Second term** captures the uncertainty of a given draw of the model parameters

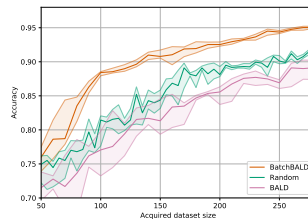
Score is high when model is uncertain in general (high entropy), but per parameter sample certain (expectation of sample entropy low).

### Batch Acquisitions

In practice, we acquire the **top-b highest scoring points**:

$$\{x_1^*, \dots, x_b^*\} = \arg \max_{\{x_1, \dots, x_b\} \subseteq \mathcal{D}_{\text{pool}}} a(\{x_1, \dots, x_b\}, \mathbf{p}(\omega | \mathcal{D}_{\text{train}}))$$

But naively applying BALD this way leads to redundant acquisitions, **under performing random acquisitions!**



Results on **Repeated MNIST**:

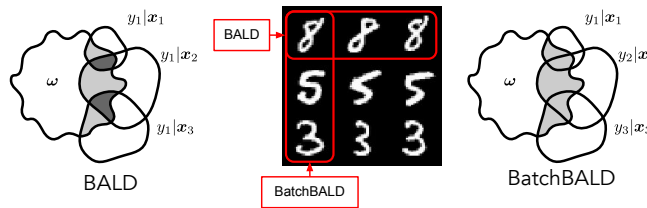
### BatchBALD

We propose to compute BALD over a **batch** of points:

$$a_{\text{BatchBALD}}(\{x_1, \dots, x_b\}, \mathbf{p}(\omega | \mathcal{D}_{\text{train}})) = \mathbb{I}(y_1, \dots, y_b; \omega | x_1, \dots, x_b, \mathcal{D}_{\text{train}})$$

Expanding the Mutual Information:

$$\mathbb{I}(y_{1:b}; \omega | x_{1:b}, \mathcal{D}_{\text{train}}) = \mathbb{H}[(y_{1:b} | x_{1:b}, \mathcal{D}_{\text{train}})] - \mathbb{E}_{\mathbf{p}(\omega | \mathcal{D}_{\text{train}})} [\mathbb{H}[(y_{1:b} | x_{1:b}, \omega, \mathcal{D}_{\text{train}})]]$$



**BALD** counts the dark areas double, while **BatchBALD** correctly computes the surface of the overlapping area

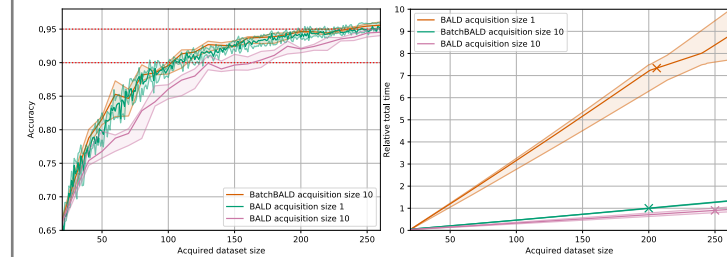
### Computing BatchBALD

Computing **joint-entropy** exact requires evaluating exponential amount of candidates. In **BatchBALD**, we compute a **greedy approximation** and build up acquisition batch one by one. We show the approximation is **submodular** with an error bounded by  $1 - \frac{1}{e}$ .

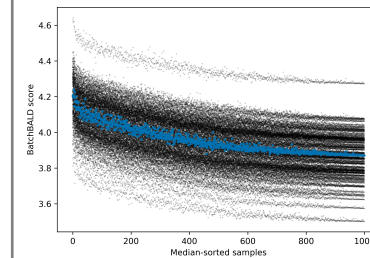
**Definition:** A function  $f$  defined on subsets of  $\Omega$  is called **submodular** if for every set  $A \subset \Omega$  and two non-identical points  $x, y \in \Omega \setminus A$ :

$$f(A \cup \{x, y\}) - f(A) \leq (f(A \cup \{x\}) - f(A)) + (f(A \cup \{y\}) - f(A))$$

### MNIST

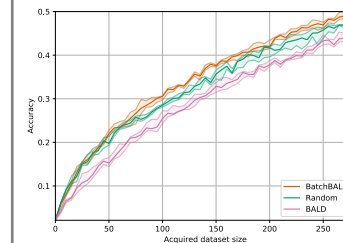


### Consistent Dropout

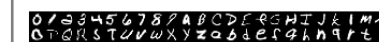


Computing BatchBALD requires keeping dropout masks constant when evaluating the acquisition score across the unlabelled pool set. As a side-effect it **reduces variance** when computing acquisition score! Also useful in BALD and other applications using dropout VI.

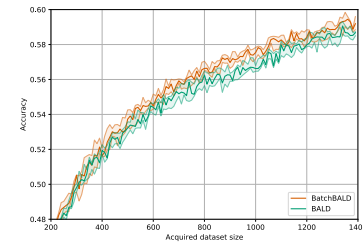
### EMNIST



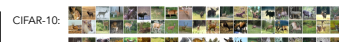
An extension of MNIST containing letters:



### CINIC-10



CINIC-10 is a combination of CIFAR and ImageNet:



[1] Houlis, Neil, et al. "Bayesian active learning for classification and preference learning." arXiv preprint arXiv:1112.5745 (2011).  
 [2] Gal, Yarin, Raashad Islam, and Zoubin Ghahramani. "Deep bayesian active learning with image data." Proceedings of the 34th International Conference on Machine Learning-Volume 70. JMLR.org, 2017.